Error Propagation of the Computed Orbital Elements of the Selected Near-Earth Asteroids

Ireneusz Wlodarczyk

MPC 553 Chorzow

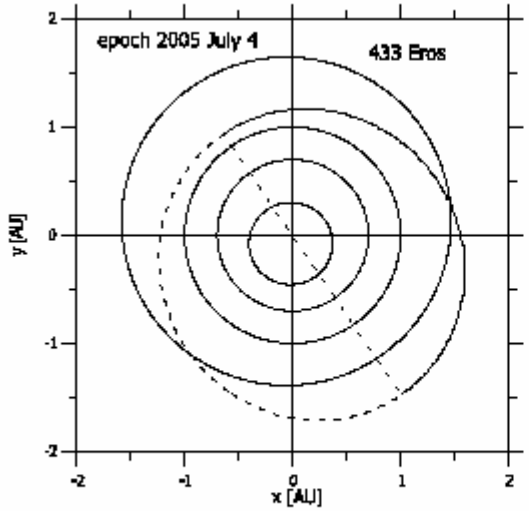
Error Propagation

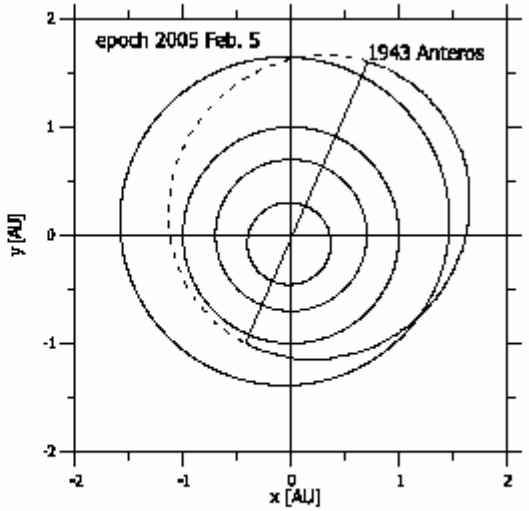
- 1. Selected asteroids
- 2. Starting errors
- 3. Propagation
- 4. Histogram of distances
- 5. Close approaches
- 6. Time of stability
- 7. Length of observational arc
- 8. Eros like Anteros
- 9. Aphelion-perihelion dependnce
- 10. For observer in the sky

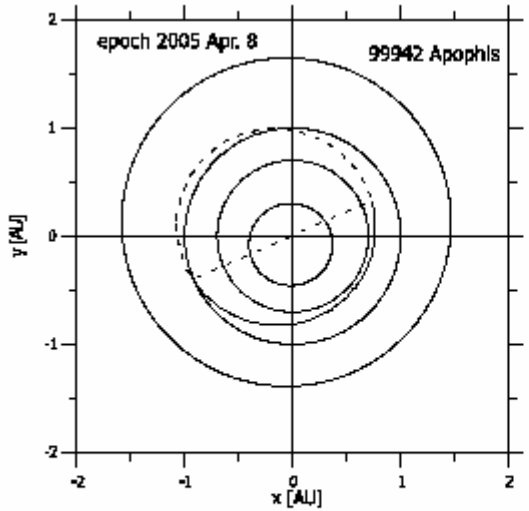
Selected NEAs

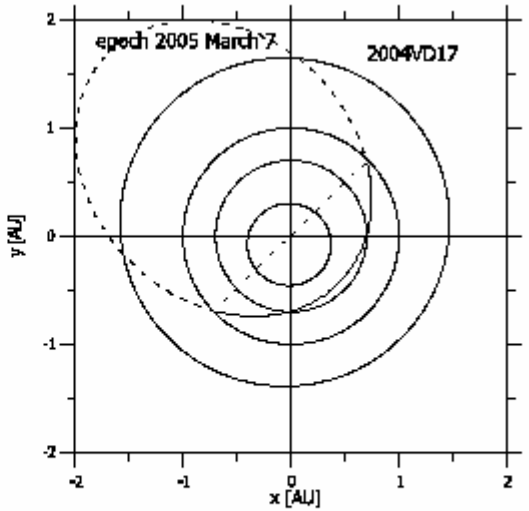
- 443 Eros – first obs. from 1893
- Anteros – discovered in 1968
- 99942 Apophis (2004MN4)
- 2004VD17

The starting orbits of the selected NEAs


epoch 2005 July 4
433 Eros
y [AU]
x [AU]


epoch 2005 Feb. 5
1943 Anteros
y [AU]
x [AU]


epoch 2005 Apr. 8
99942 Apophis
y [AU]
x [AU]


epoch 2005 March 7
2004VD17
y [AU]
x [AU]

Methods of omputations

- Mercury package (Chambers, 1999).
- Planetary initial ephemeris: *DE405/WAW (Sitarski, 2002)*.

The influence of the epoch of the computed orbital elements

- The starting orbital elements of the selected clones of asteroids for the epoch of the beginning, middle and the end of the observational arc were computed by Prof. Grzegorz Sitarski from the Space Researcg Center of the Polish Academy of Sciences in Warsaw

Table 1

The starting nominal orbital elements of the selected asteroids

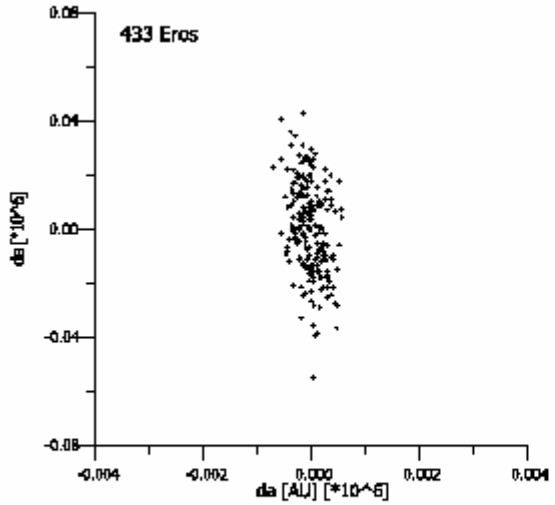
N_7	M	a[AU]	e	ω_{2000}	Ω_{2000}	i_{2000}
433 Eros - 5274 observations from 40 789 days (1893 Oct. 29 - 2005 Jul. 3), rms=0.82"						
The nominal orbit for the beginning of the observational arc: epoch 1893 Oct. 24.0						
1	309°.15040460	1.45804369260	0.22275909889	177°.49510835	305°.00161542	10°.83462251
The nominal orbit for the middle of the observational arc: epoch 1950 Jan. 19.0						
2	285°.89247718	1.45819437488	0.22300270701	178°.16519743	304°.69561818	10°.83397842
The nominal orbit for the end of the observational arc: epoch 2005 Jul. 04.0						
3	102°.95189106	1.45812119024	0.22277396063	178°.67137896	304°.39109031	10°.82896571
1943 Anteros - 775 observations from 11 654 days (1973 Mar. 10 - 2005 Feb. 4)						
The nominal orbit for the beginning of the observational arc: epoch 1973 Mar. 10.0						
1	330°.43770026	1.43141137237	0.25645967085	337°.89421188	246°.63835595	8°.69534146
The nominal orbit for the middle of the observational arc: epoch 1989 Feb. 21.0						
2	86°.97996637	1.43012411552	0.25608357912	338°.11290413	246°.50234436	8°.70342010
The nominal orbit for the end of the observational arc: epoch 2005 Feb. 5.0						
3	205°.18118558	1.43026762744	0.25592019902	338°.24348827	246°.40866106	8°.70414752
99942 Apophis - 884 observations from 389 days (2004 Mar. 15 - 2005 Apr. 8)						
The nominal orbit for the beginning of the observational arc: epoch 2004 Mar. 15.0						
1	139°.96836059	0.92197289	0.19118625	126°.17021845	204°.57875231	3°.33338897
The nominal orbit for the middle of the observational arc: epoch 2004 Oct. 1.0						
2	2°.62780888	0.92196629	0.19118743	126°.18225693	204°.57563013	3°.33358226
The nominal orbit for the end of the observational arc: epoch 2005 Apr. 8.0						
3	212°.87561054	0.92239674	0.19103240	126°.38279763	204°.47182960	3°.33097679
2004 VD17 - 720 observations from 117 days (2004 Nov. 7 - 2005 Mar. 4), rms=0.42"						
The nominal orbit for the beginning of the observational arc: epoch 2004 Nov. 07.0						
1	29°.35446892	1.50828865	0.58881659	90°.70558763	224°.25113177	4°.22303575
The nominal orbit for the middle of the observational arc: epoch 2005 Jan. 07.0						
2	61°.83101214	1.50801549	0.58875944	90°.69809952	224°.25044164	4°.22284415
The nominal orbit for the end of the observational arc: epoch 2005 Mar. 07.0						
3	93°.23498004	1.50799076	0.58876064	90°.69693025	224°.25019170	4°.22283177

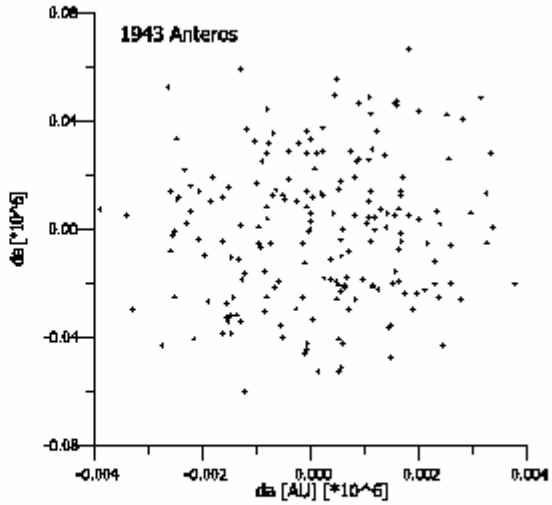
where M - mean anomaly, a - semimajor axis, e - eccentricity, ω_{2000} - argument of perihelion, Ω_{2000} - longitude of the ascending node, i_{2000} - inclination of the orbit. These orbital elements are referred to the J2000 equator and equinox.

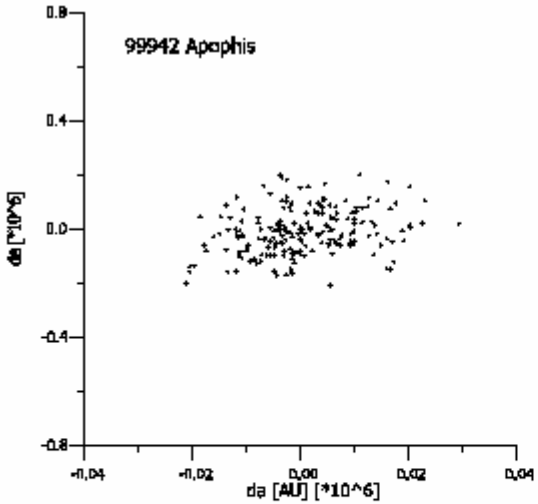
The starting ellipsoids of the errors of the orbital elements for the epoch of the end of the observational arc

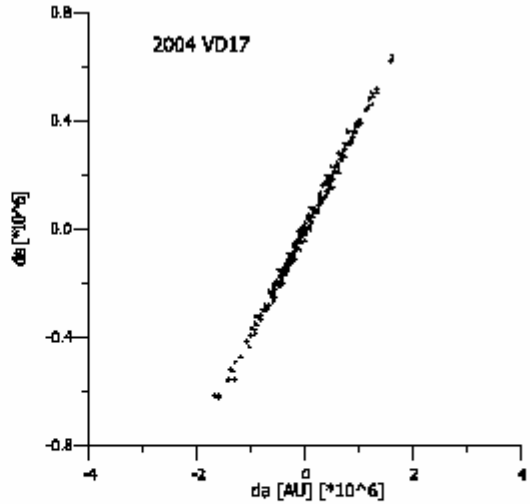
The computed from observations orbital elements of asteroids contain errors

Fig. shows the starting ellipsoid of the errors of the clones of the four selected asteroids – in a, e: differences beween nominal ,orbit and clones (x 10^6)


433 Eros
0.08
0.04
0.00
-0.04
-0.08
da [AU] [*10^-6]
-0.004
-0.002
0.000
0.002
0.004


1943 Anteros
0.08
0.04
0.00
-0.04
-0.08
da [AU] [*10^-6]
-0.004
-0.002
0.000
0.002
0.004


99942 Apophis
0.8
0.4
0.0
-0.4
-0.8
da [AU] [*10^-6]
-0,04
-0,02
0,00
0,02
0,04


2004 VD17
0.8
0.4
0.0
-0.4
-0.8
da [AU] [*10^-6]
-4
-2
0
2
4

Test of computations

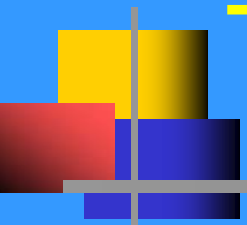
The equations of motion of the nominal orbit of Eros were integrated 10,000 years and 20,000 years forwards and then backwards to the starting epoch

Table 2

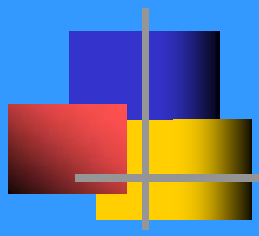
433 Eros - The results of the forward and backward integration of the nominal orbit of Eros

	M	$a[AU]$	e	ω_{2000}	Ω_{2000}	i_{2000}
The nominal orbit for the beginning of the observational arc: epoch 1893 Oct. 24.0 (JD 2,412,760.5)						
start	309°.1504046	1.4580436926	0.22275909889	177°.49510835	305°.00161542	10°.83462251
10,000 years	309°.1479904	1.4580436917	0.22275909967	177°.49510835	305°.00161576	10°.83462249
20,000 years	309°.1463082	1.4580436913	0.22275910024	177°.49510835	305°.00161600	10°.83462248

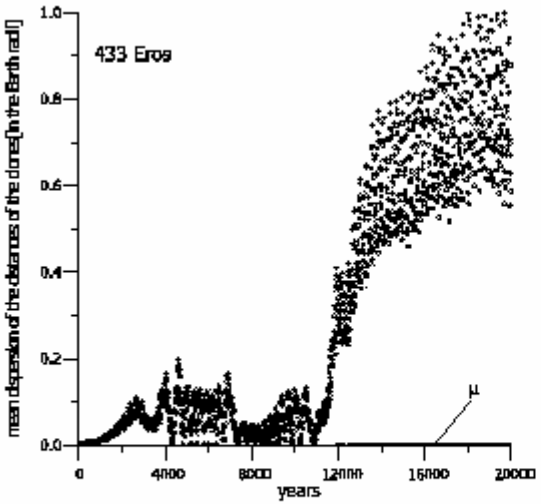
where M - mean anomaly, a - semimajor axis, e - eccentricity, ω_{2000} - argument of perihelion, Ω_{2000} - longitude of the ascending node, i_{2000} - inclination of the orbit. These orbital elements refer to the J2000 equator and equinox.

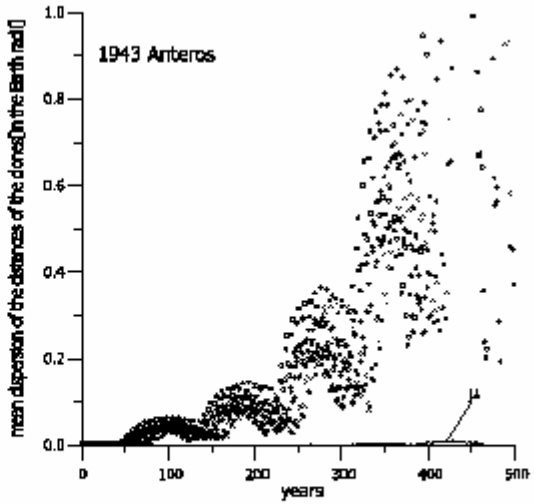


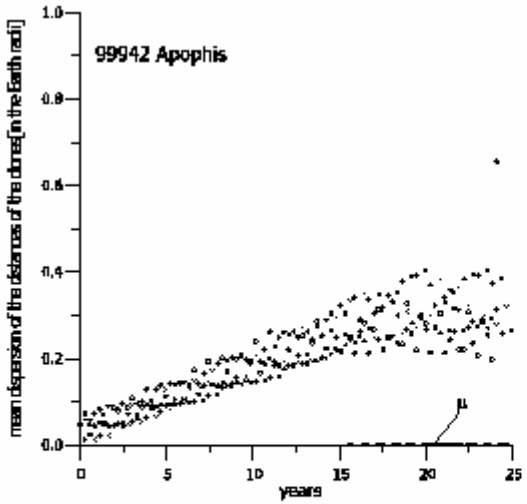
The propagation of the errors

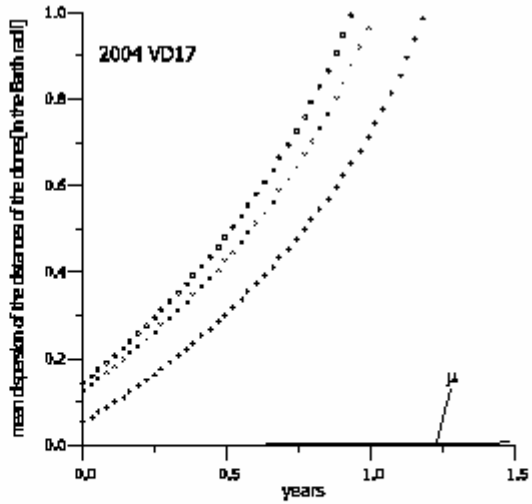


- The computations were made for different epochs
- The computations were stopped when the mean distances between nominal orbit and clones exceed one Earth radius
- The error depends on the length of the observational length

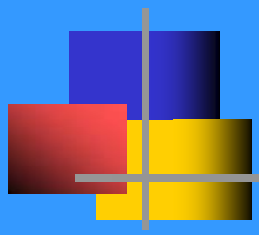

433 Eros
mean dispersion of the distances of the clones [in the Earth radii]
years
μ


1943 Anteros
mean dispersion of the distances of the clones [in the Earth radii]
years
μ


99942 Apophis
mean dispersion of the distances of the clones [in the Earth radii]
years
μ

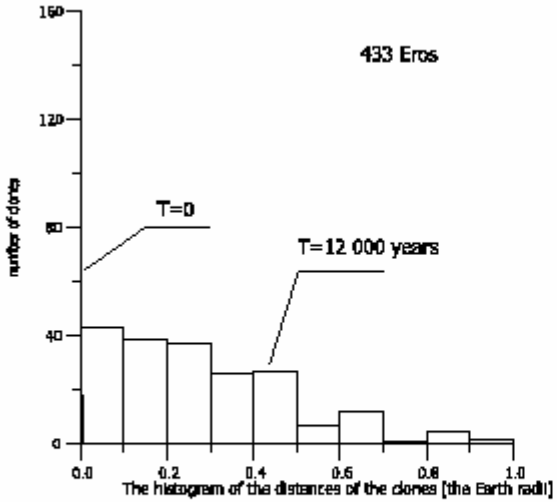

2004 VD17
mean dispersion of the distances of the clones [in the Earth radii]
years
μ

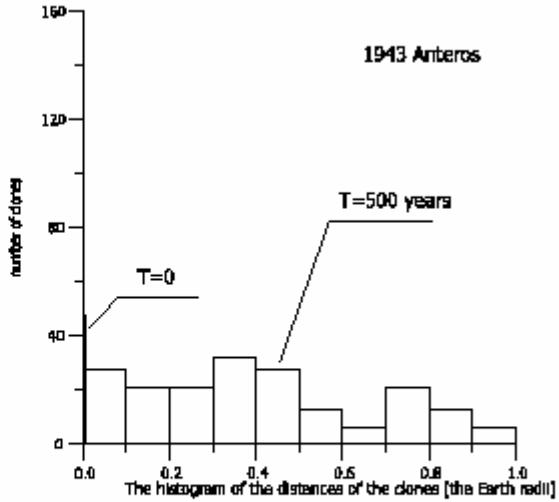
The histograms of the temporary distances of the VA

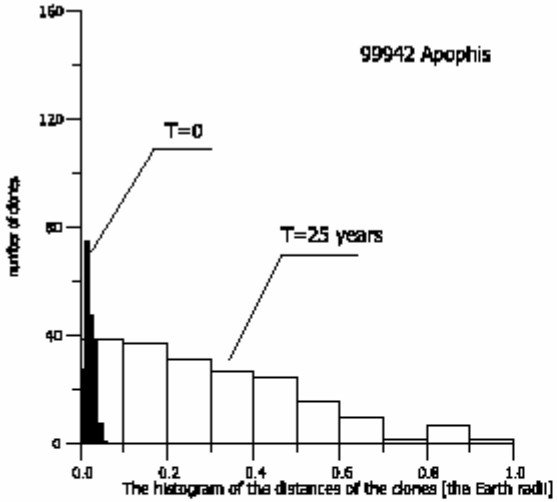


- Eros – 12,000 years – all clones are inside the torus, after 16,000 years – 128
- Anteros – 500 years – 189
- Apophis – 25 years – 196
- 2004VD17 – 1.2 years – 1/3 only

  (out of 199 starting everywhere)


433 Eros
T=0
T=12 000 years
number of clones
The histogram of the distances of the clones [the Earth radii]


1943 Anteros
T=0
T=500 years
number of clones
The histogram of the distances of the clones [the Earth radii]


99942 Apophis
T=0
T=25 years
number of clones
The histogram of the distances of the clones [the Earth radii]

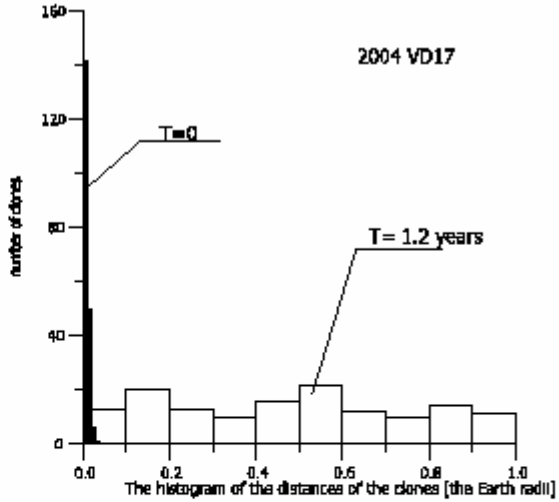
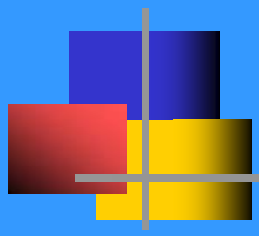

2004 VD17
T=0
T= 1.2 years
number of clones
The histogram of the distances of the clones [the Earth radii]

Table 3

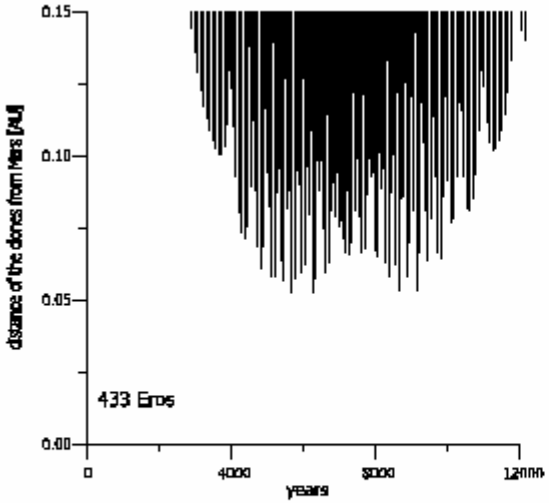
433 Eros - The time evolution of the temporary distances of the clones from the nominal orbit

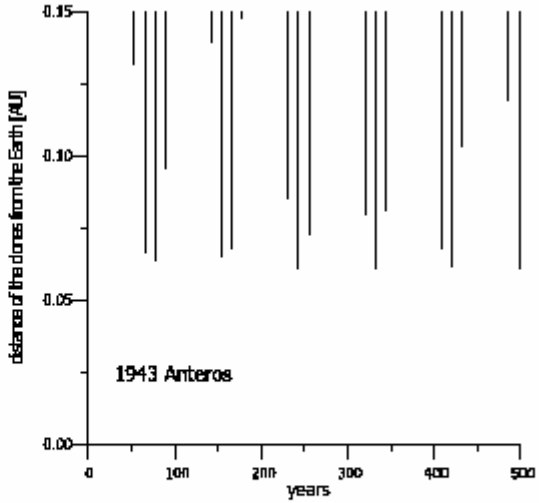
time [kyrs]	The number of clones in the intervals of the distance [AU]										
	0− −0.1	0.1− −0.2	0.2− −0.3	0.3− −0.4	0.4− −0.5	0.5− −0.6	0.6− −0.7	0.7− −0.8	0.8− −0.9	0.9− −1.0	sum
start	199										199
1	199										199
2	193	6									199
5	121	62	16								199
12	43	39	37	26	27	7	12	1	5	2	199
16	13	13	15	12	17	11	10	14	15	8	128

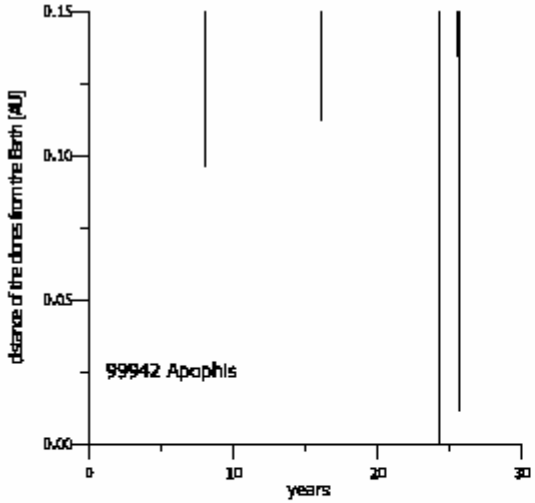
The close approaches to planets

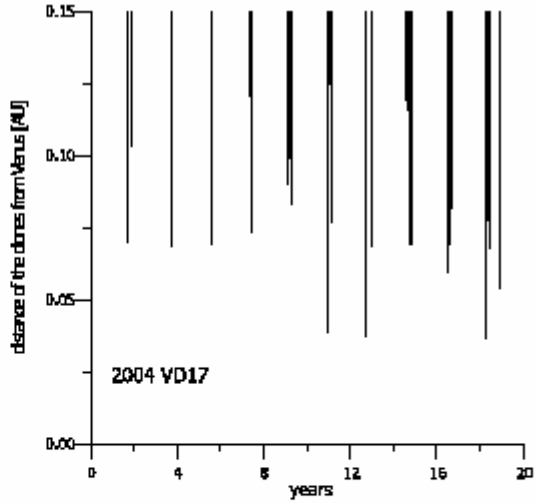


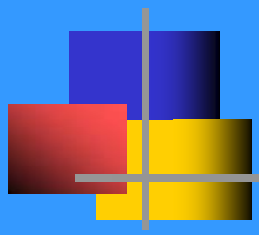
- The closer the approaches to the planets the more chaotic the orbit


0.15
0.10
0.05
0.00
distance of the clones from Mars [AU]
433 Eros
0
4000
8000
12000
years


0.15
0.10
0.05
0.00
distance of the clones from the Earth [AU]
1943 Anteros
0
100
200
300
400
500
years


0.15
0.10
0.05
0.00
distance of the clones from the Earth [AU]
99942 Apophis
0
10
20
30
years


0.15
0.10
0.05
0.00
distance of the clones from Venus [AU]
2004 VD17
0
4
8
12
16
20
years

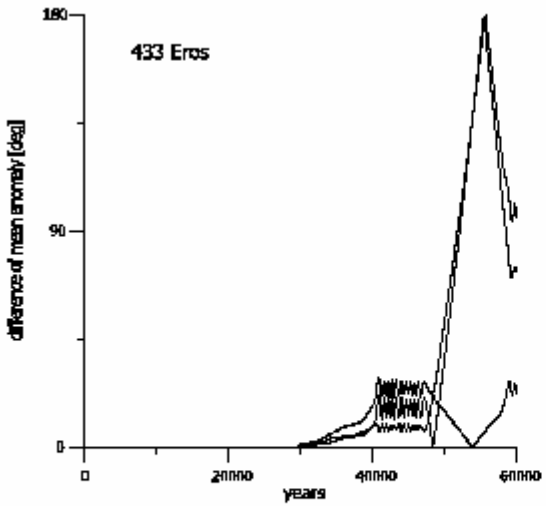


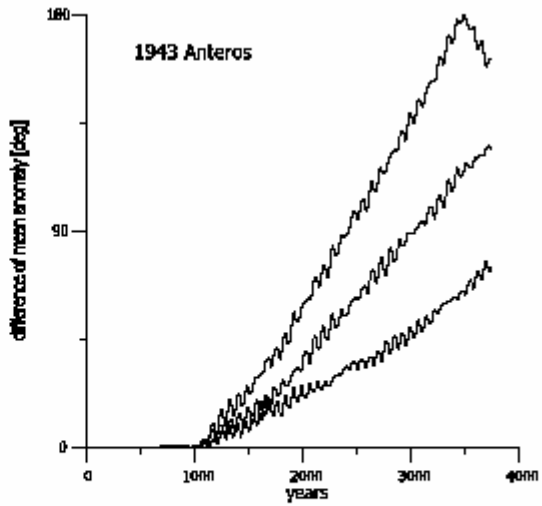
- Anteros has close approaches to the Earth which has a greater mass than that of Mars
- The orbit of Anteros is more tangential to the orbit of the Earth (i=8.7 and i=10.8, respectively)

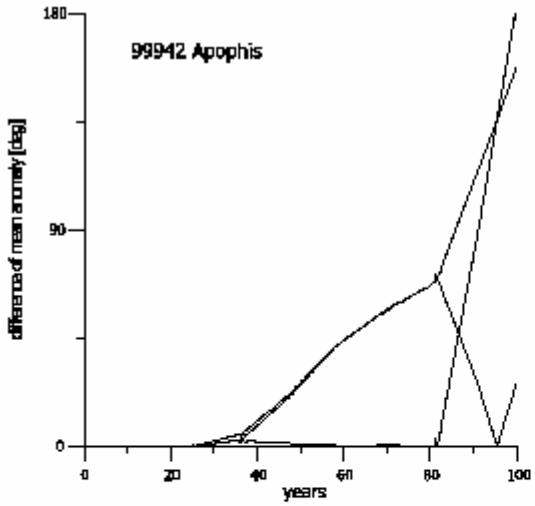
The time of stability

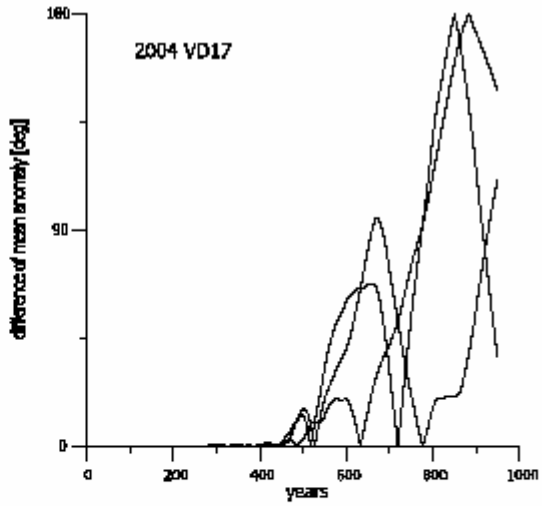
Time of stability

- Is the time after which we can observe rapid growth of the differences in the mean anomaly of the neighboring clones
- Computations were stoped when dM reached 180 deg.

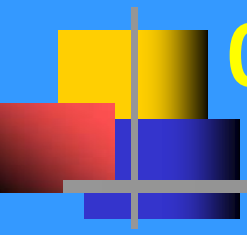

433 Eros
difference of mean anomaly [deg]
180
90
0
0
20000
40000
60000
years

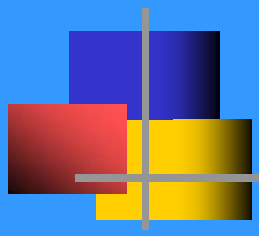

1943 Anteros
difference of mean anomaly [deg]
180
90
0
0
1000
2000
3000
4000
years


99942 Apophis
difference of mean anomaly [deg]
180
90
0
0
20
40
60
80
100
years


2004 VD17
difference of mean anomaly [deg]
180
90
0
0
200
400
600
800
1000
years

Computed orbits from different observational arcs



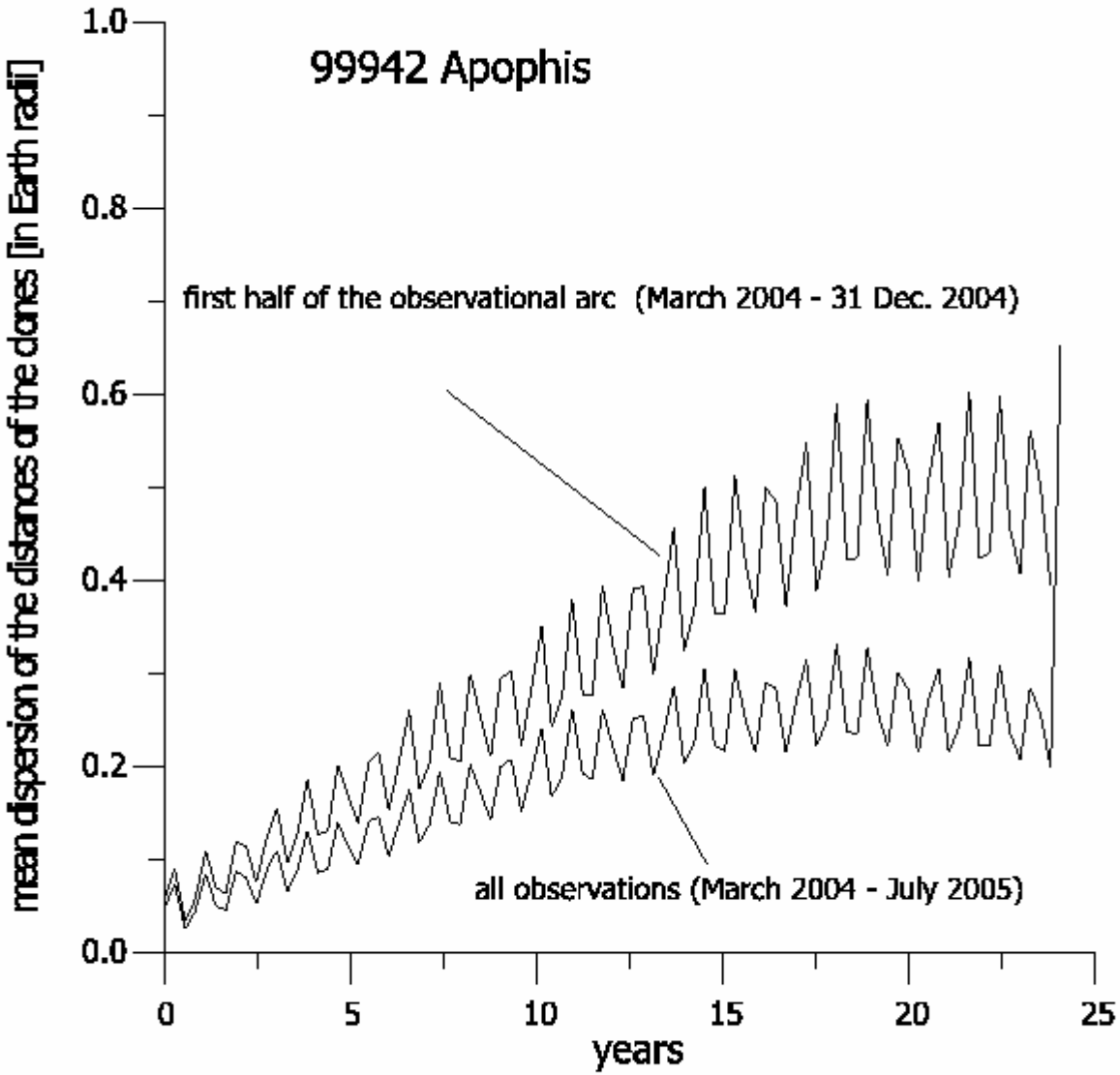


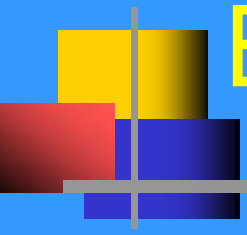
- Propagation of the clones depends on the length of the observational arc – example of Apophis

Table 4

99942 Apophis - orbits computed using different observational arcs

Nr	M	$a[AU]$	e	ω_{2000}	Ω_{2000}	i_{2000}
99942 Apophis - 884 observations from 389 days (2004 Mar. 15 - 2005 Apr. 8), rms=0.46"						
The orbit for the epoch 2005 Apr. 8.0						
1	212°.875611054	0.92239674	0.19103240	126°.38279763	204°.47182960	3°.33097679
99942 Apophis - 329 observations (2004 Mar. 15 - 2004 Dec. 31)						
The orbit for the epoch 2005 Apr. 8.0						
2	212°.87599812	0.92239616	0.19103212	126°.38193365	204°.47251401	3°.33096826


99942 Apophis
first half of the observational arc (March 2004 - 31 Dec. 2004)
all observations (March 2004 - July 2005)
mean dispersion of the distances of the clones [in Earth radii]
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
years



Eros like Anteros

Eros like Anteros

- Eros and Anteros have similar orbits
- To eliminate the influence of the observational arc and the number of the observation of these asteroids a selection of their observations was made

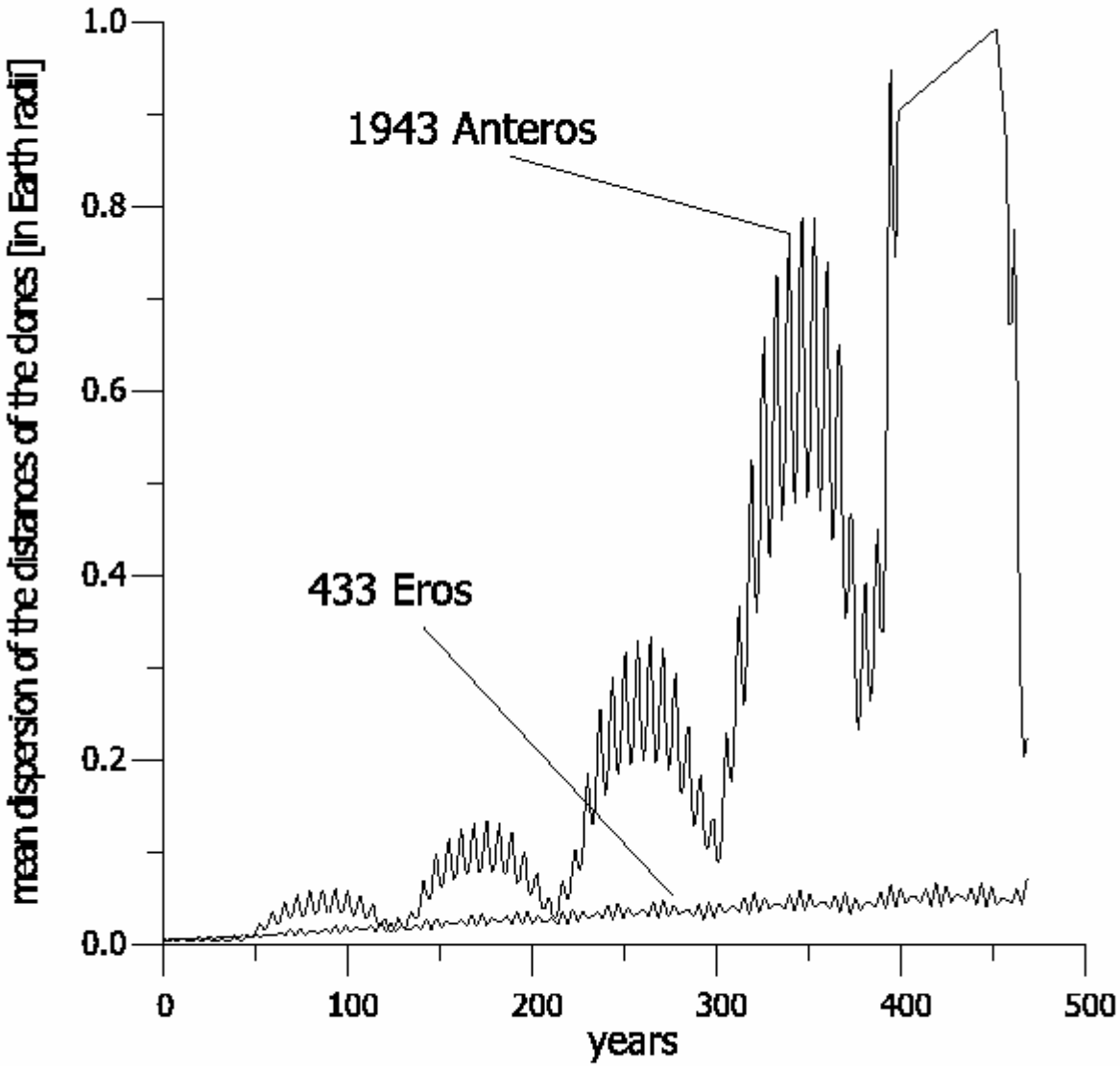
Eros like Anteros

- The observations of Eros were chosen only from the time-span of the period of the observations of Anteros: 1973-2005
- Every fourth observation of Eros was selected: hence we obtained 763 observations of Eros and 775 observations of Anteros

Table 5

The starting nominal orbital elements of Eros and Anteros normalized to observational arc of Anteros

Nr	M	$a[AU]$	e	ω_{2000}	Ω_{2000}	i_{2000}
The orbit of Anteros for the end of the observations arc: epoch 2005 Feb. 5.0						
1	205°.18118558	1.43026762744	0.25592019902	338°.24348827	246°.40866106	8°.70414752
The orbit of Eros: epoch 2005 Mar. 9.0						
1	37°.45013740	1.45818348264	0.22277025076	178°.67433495	304°.39910374	10°.82906384


1943 Anteros
433 Eros
mean dispersion of the distances of the clones [in Earth radii]
1.0
0.8
0.6
0.4
0.2
0.0
0
100
200
300
400
500
years

Eros like Anteros

- Cause of the differences in the propagation of the clones of Eros and Anteros are different close approaches to planets

Problem aphelion-perihelion

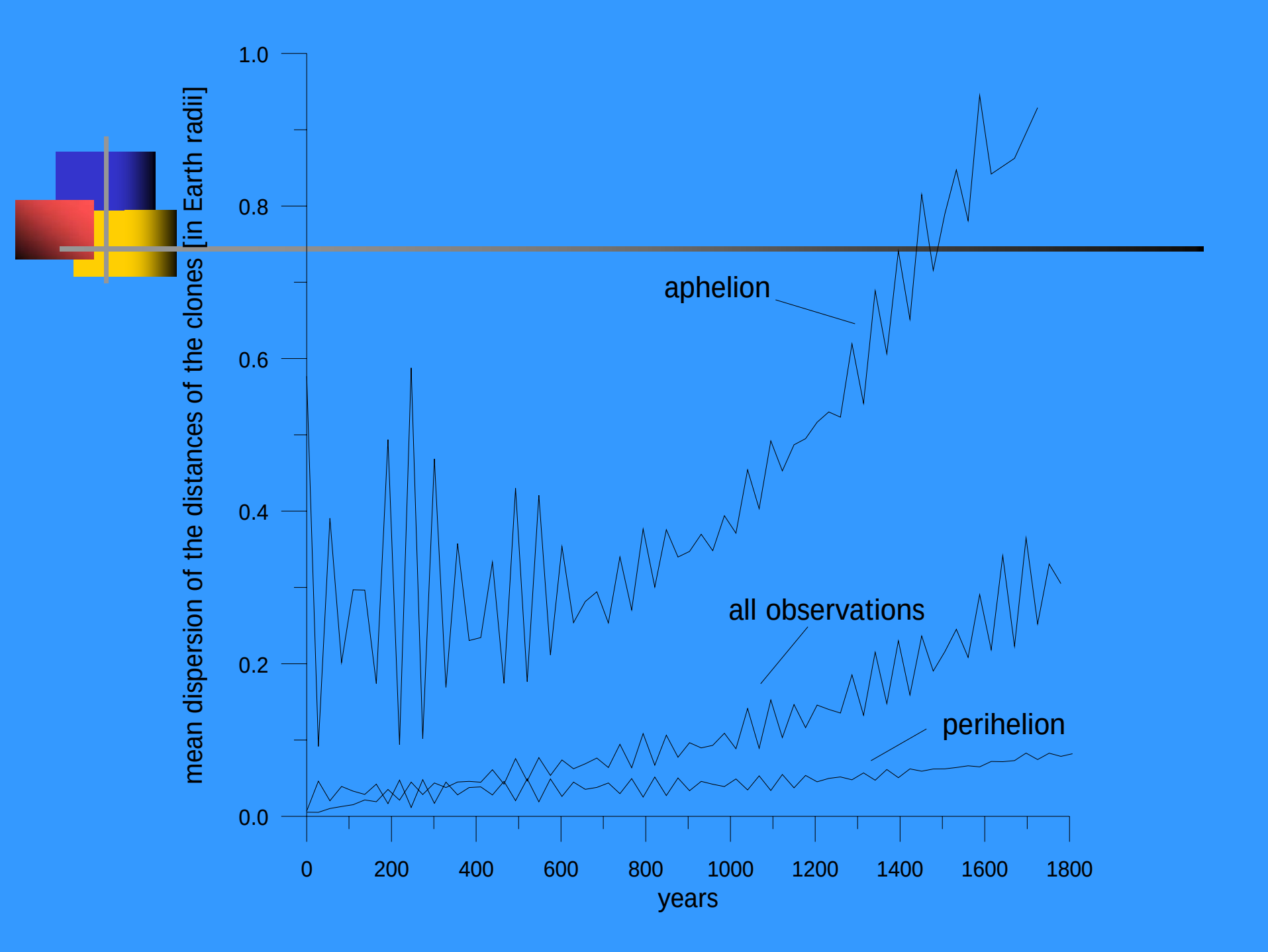
Aphelion-perihelion (Eros)

- Observations selected around aphelion and perihelion: +-10 deg.
- In aphelia clones are 6 times more divergent then in perihelia and four times than those from all the observations
- In order to compute the real orbit we must use all the observations

Table 6

The starting orbital elements of the nominal orbit of the Eros computed from the observations made near the Eros's aphelia and near the Eros's perihelia

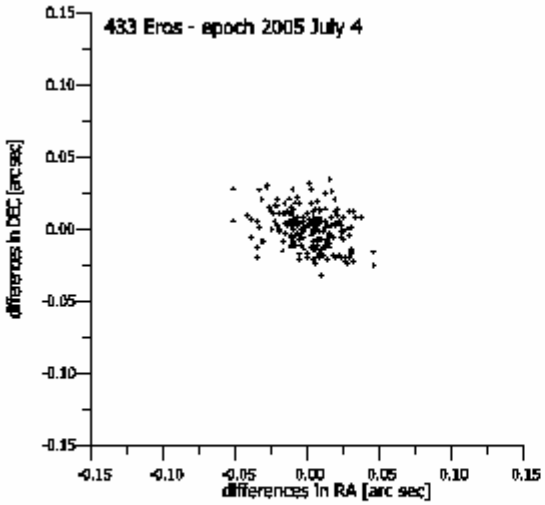
Nr	M	$a[AU]$	e	ω_{2000}	Ω_{2000}	i_{2000}
433 Eros - the observations near the aphelia						
The orbit for the epoch 2003 May 1.0						
1	17°.99444344	1.45828828433	0.22294154341	178°.65211672	304°.40466440	10°.83011320
433 Eros - the observations near the perihelia						
The orbit for the epoch 2003 May 1.0						
2	17°.99251353	1.45828827764	0.22293875013	178°.65330215	304°.40473481	10°.83004489

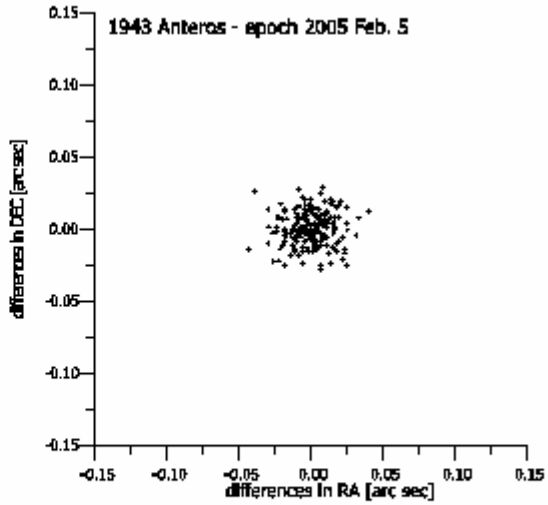

mean dispersion of the distances of the clones [in Earth radii]
1.0
0.8
0.6
0.4
0.2
0.0
aphelion
all observations
perihelion
0
200
400
600
800
1000
1200
1400
1600
1800
years

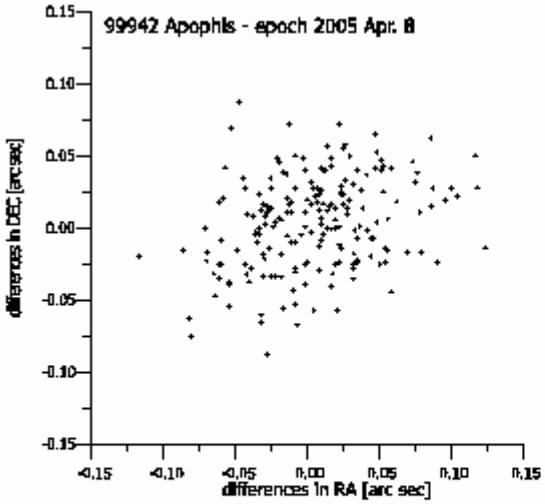
The errors in the sky plane

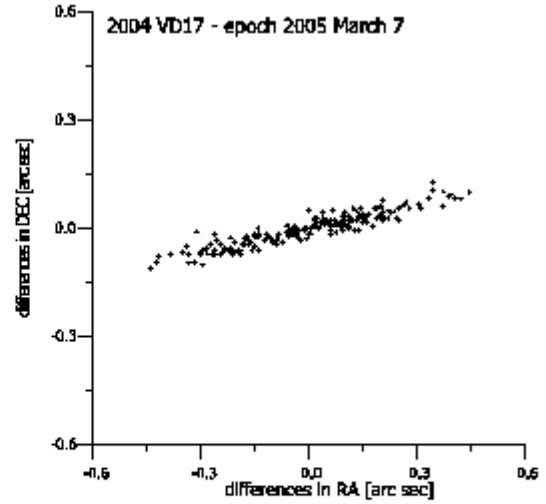
For Observers

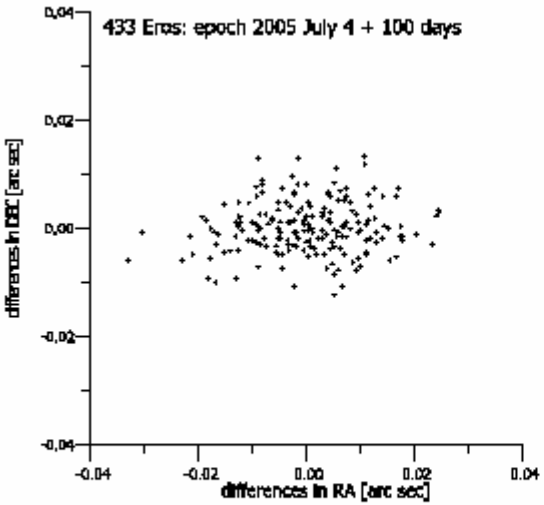
- Each of the computed clones of the asteroid is located in a certain distance from the asteroid on the nominal orbit
- Fig. presents the region in the sky occupied by clones for a given epoch of orbital elements – similar to „Uncertainty Region (Confidence Boundary)” at Milani NEODYS site

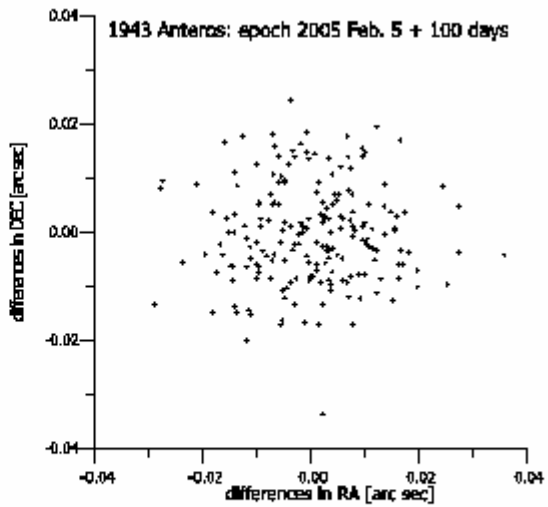

433 Eros - epoch 2005 July 4
differences in DEC [arc sec]
differences in RA [arc sec]
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15

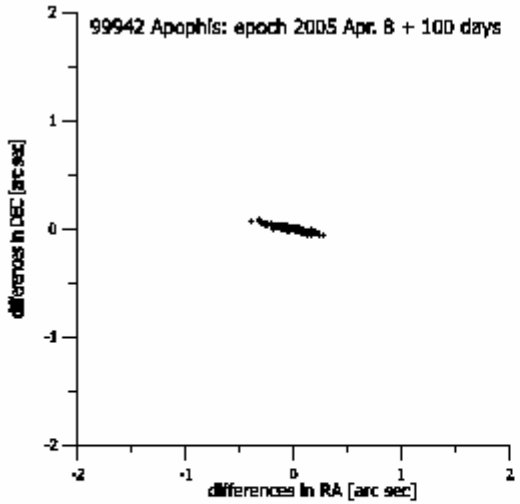

1943 Anteros - epoch 2005 Feb. 5
differences in DEC [arc sec]
differences in RA [arc sec]
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15

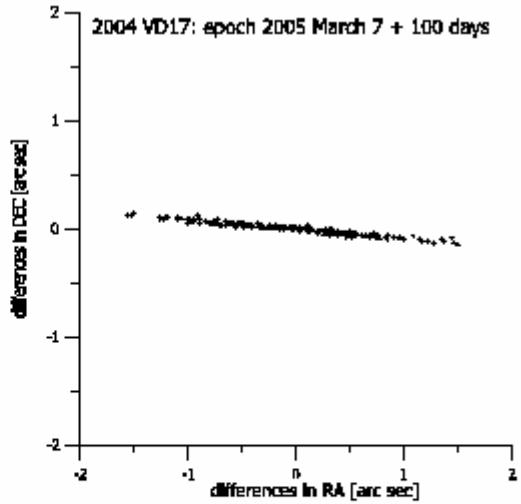

99942 Apophis - epoch 2005 Apr. 8
differences in DEC [arc sec]
differences in RA [arc sec]
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15

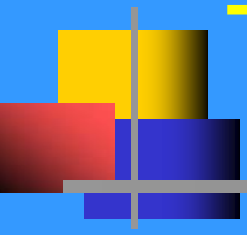

2004 VD17 - epoch 2005 March 7
differences in DEC [arc sec]
differences in RA [arc sec]
0.6
0.3
0.0
-0.3
-0.6


433 Eros: epoch 2005 July 4 + 100 days
differences in DEC [arc sec]
differences in RA [arc sec]
0.04
0.02
0.00
-0.02
-0.04


1943 Anteros: epoch 2005 Feb. 5 + 100 days
differences in DEC [arc sec]
differences in RA [arc sec]
0.04
0.02
0.00
-0.02
-0.04


99942 Apophis: epoch 2005 Apr. 8 + 100 days
differences in DEC [arc sec]
differences in RA [arc sec]
2
1
0
-1
-2


2004 VD17: epoch 2005 March 7 + 100 days
differences in DEC [arc sec]
differences in RA [arc sec]
2
1
0
-1
-2



Thank you